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## Abstract:

Ice shelves evolve under the influence of many factors, the main ones being melting at the grounding line, refreezing downwards of the grounding line and up to the ice front, melting and calving at the ice front, and of course, ice flow under its own weight. In addition, ice shelf flow is often undercut by the presence of rifts, i.e. cracks that penetrate the entire ice shelf thickness and that are filled with seawater or ice melange, that perturb the flow significantly. Satellite radar interferometry observations of LarsenC, Larsen B and the Brunt ice-shelf also reveal the presence of faults, i.e. cracks with shear stress along the flanks of the rift, that have a significant impact on ice shelf flow, and yet differ completely from rifts.

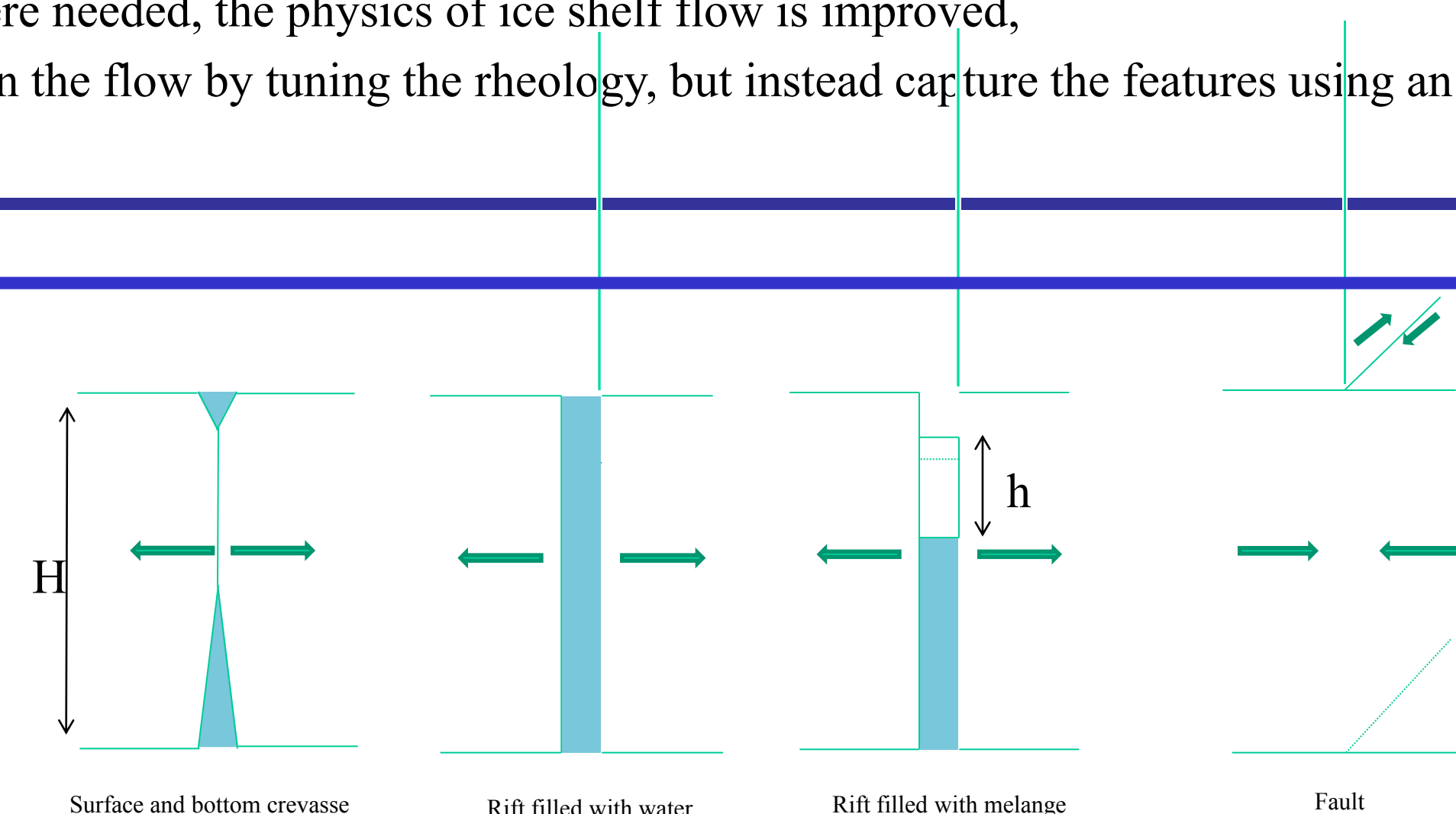
Inverse control methods have been applied to ice shelves (Rommelaere 1997, Larour 2005, Vieli 2006, Khazendar 2007) to infer the unknown rheology, but these methods rely on continuum mechanics. One of the consequences is that weak rheology is systematically inferred wherever faults are rifts or present. This issue in inverse control methods precludes any quantitative use of the inferred rheologies.

This work presents a new formulation, based on penalty methods, that can model the behavior of rifts and faults (with shear stress along the flanks of rifts, opening rates, and fill-in by melange), and that can be integrated in current inverse control methods. We use ISSM (Ice Sheet System Model), developed at JPL/UCI to model thermal-mechanical steady-state and transient flow, to implement this new formulation. We apply the improved inverse control methods on several key ice shelves in the

Antarctic Peninsula, and account for the main rifts and faults. The model is capable of replicating features in the ice flow that up to now were never captured, and it improves data assimilation results by correctly capturing the discontinuous mechanics of rifts, therefore providing a more realistic map of rheology. By including rifts and faults where needed, the physics of ice shelf flow is improved, as the models do not try to accommodate for unknown features in the flow by tuning the rheology, but instead capture the features using an improved formulation of fracture.

Several types of rifts/faults can be observed in ice shelves:

- surface and bottom crevasses in the process of creating rifts.
- rifts filled with water or melange: opening flanks on both sides of the rift.
- fault: rift that stopped opening and is closing, with contact between both flanks, involving friction.



Boundary conditions for opening rift, with arbitrary melange height  $h$ , fraction of thickness  $H$  between 0 (full of water) and 1 (filled with ice”).

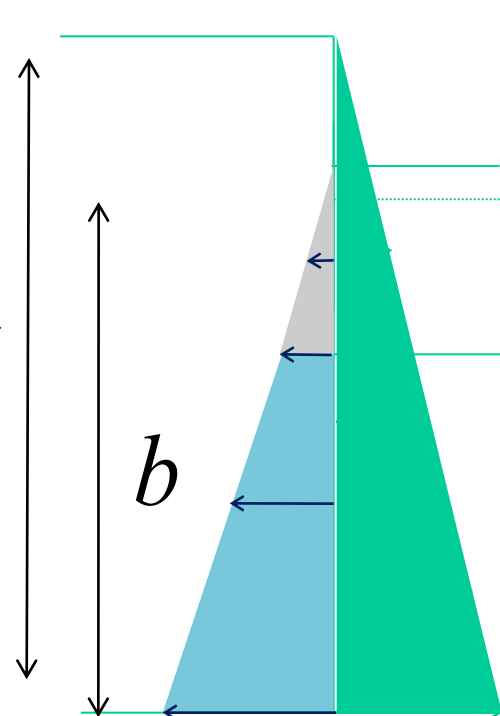
$$P = P_{\text{litho}} - P_{\text{air}} - P_{\text{melange}} - P_{\text{water}}$$

$$P_{\text{litho}} = \frac{\rho_{\text{ice}} g H^2}{2}$$

$$P_{\text{water}} = \frac{\rho_{\text{water}} g \left[ 1 - \left( \frac{\rho_{\text{ice}}}{\rho_{\text{water}}} \alpha H \right)^2 \right]}{2} H$$

$$P_{\text{melange}} = \frac{\rho_{\text{ice}} g (\alpha H)^2}{2}$$

$$P_{\text{air}} = 0$$



$$h = \alpha H$$

Boundary conditions for closing fault:

Penalized normal penetration:

$$\sigma_n = -K_{\text{max}} 10^2 \Delta V_n$$

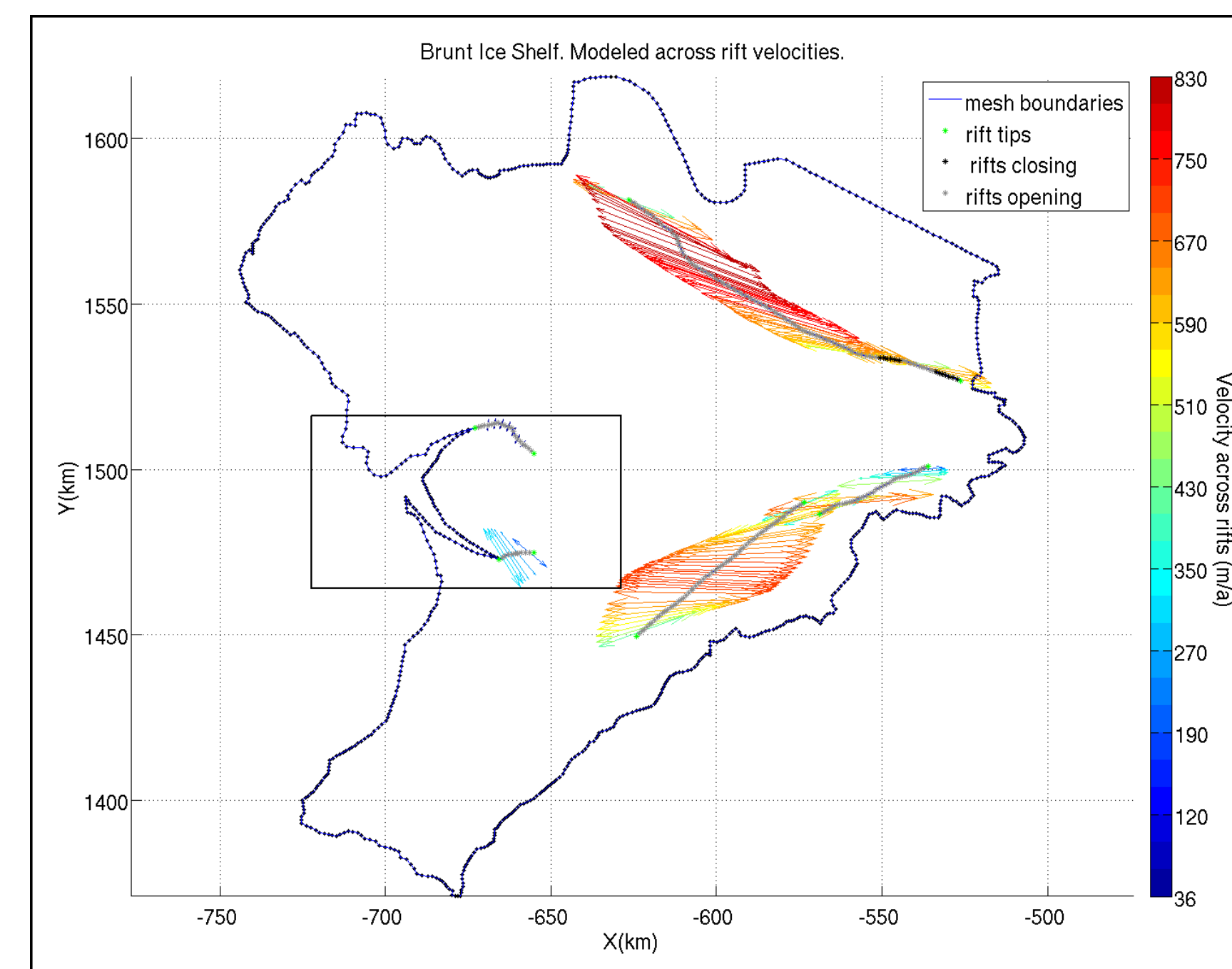
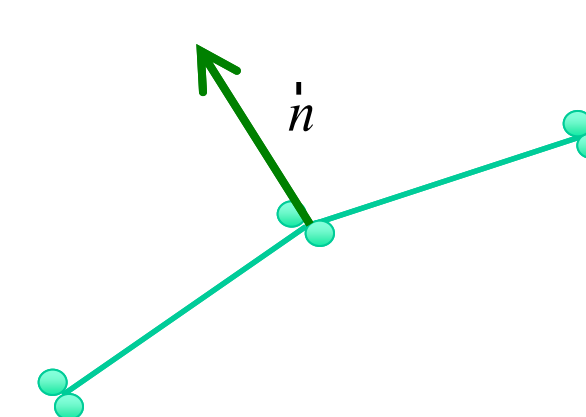
$$Kn = K_{\text{max}} 10^2 \begin{bmatrix} n_x^2 & n_x n_y & -n_x^2 & -n_x n_y \\ n_x n_y & n_y^2 & -n_x n_y & -n_y^2 \\ -n_x^2 & -n_x n_y & n_x^2 & n_x n_y \\ n_x n_y & -n_y^2 & n_x n_y & n_y^2 \end{bmatrix}$$

Tangential linear friction:

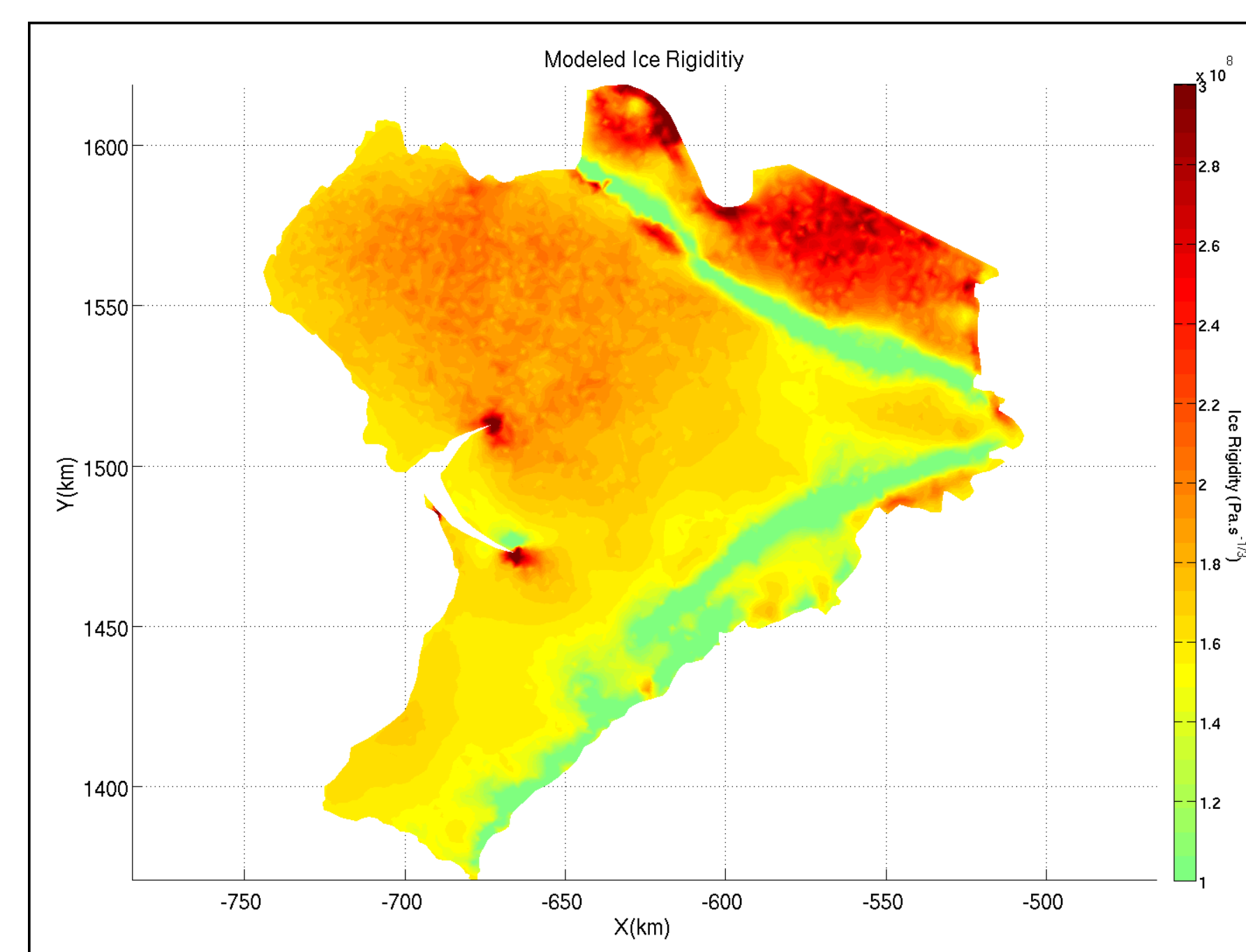
$$\sigma_t = f * H * l * \Delta V_t$$

$$Kt = H * l * f \begin{bmatrix} n_y^2 & -n_x n_y & -n_y^2 & n_x n_y \\ n_x n_y & n_x^2 & n_x n_y & -n_x^2 \\ -n_y^2 & n_x n_y & n_y^2 & -n_x n_y \\ n_x n_y & -n_x^2 & -n_x n_y & n_x^2 \end{bmatrix}$$

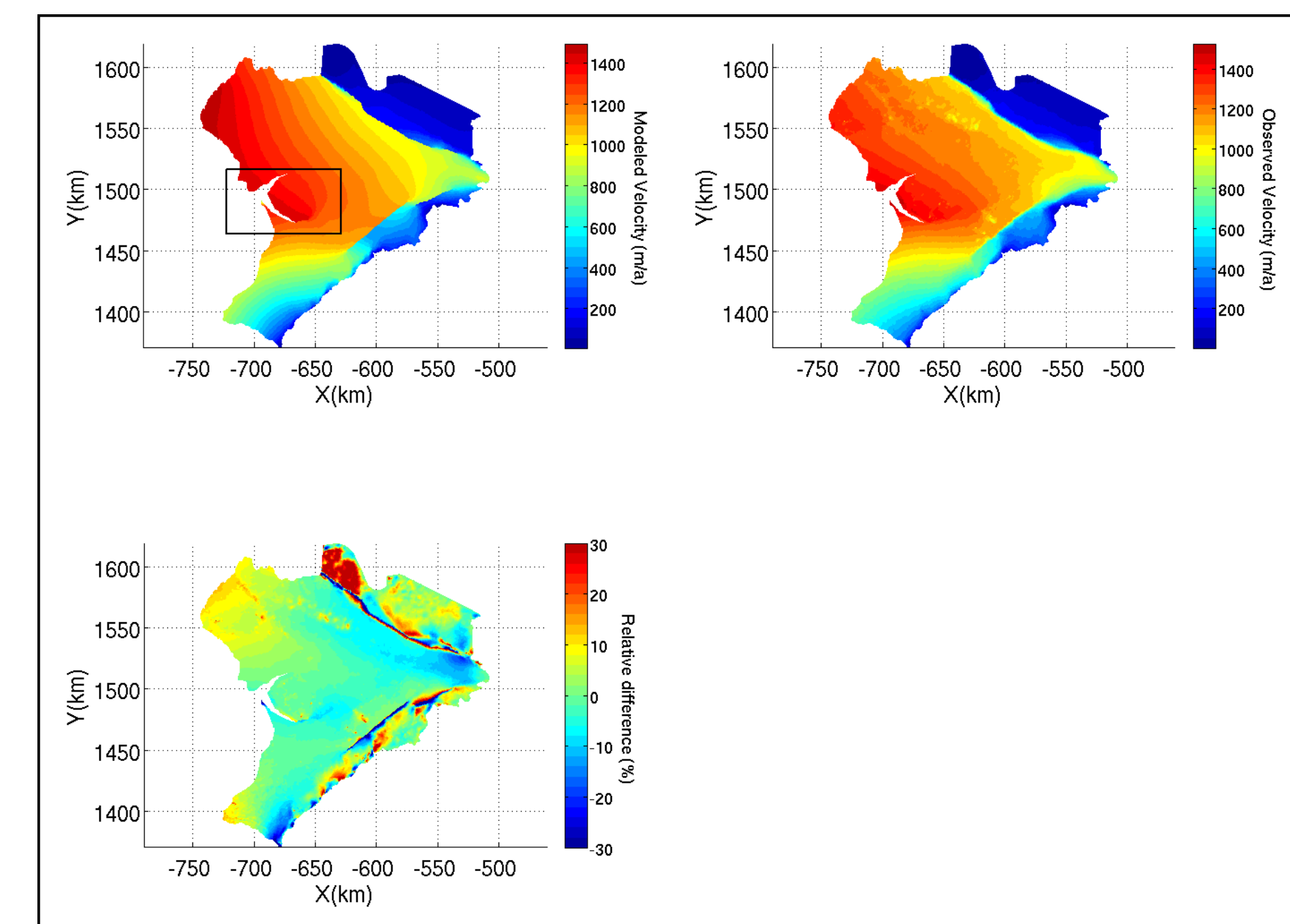
Kmax: maximum stiffness matrix  
 $\lambda$ : penalty offset.  
H: ice thickness  
l: segment length  
f: friction coefficient.



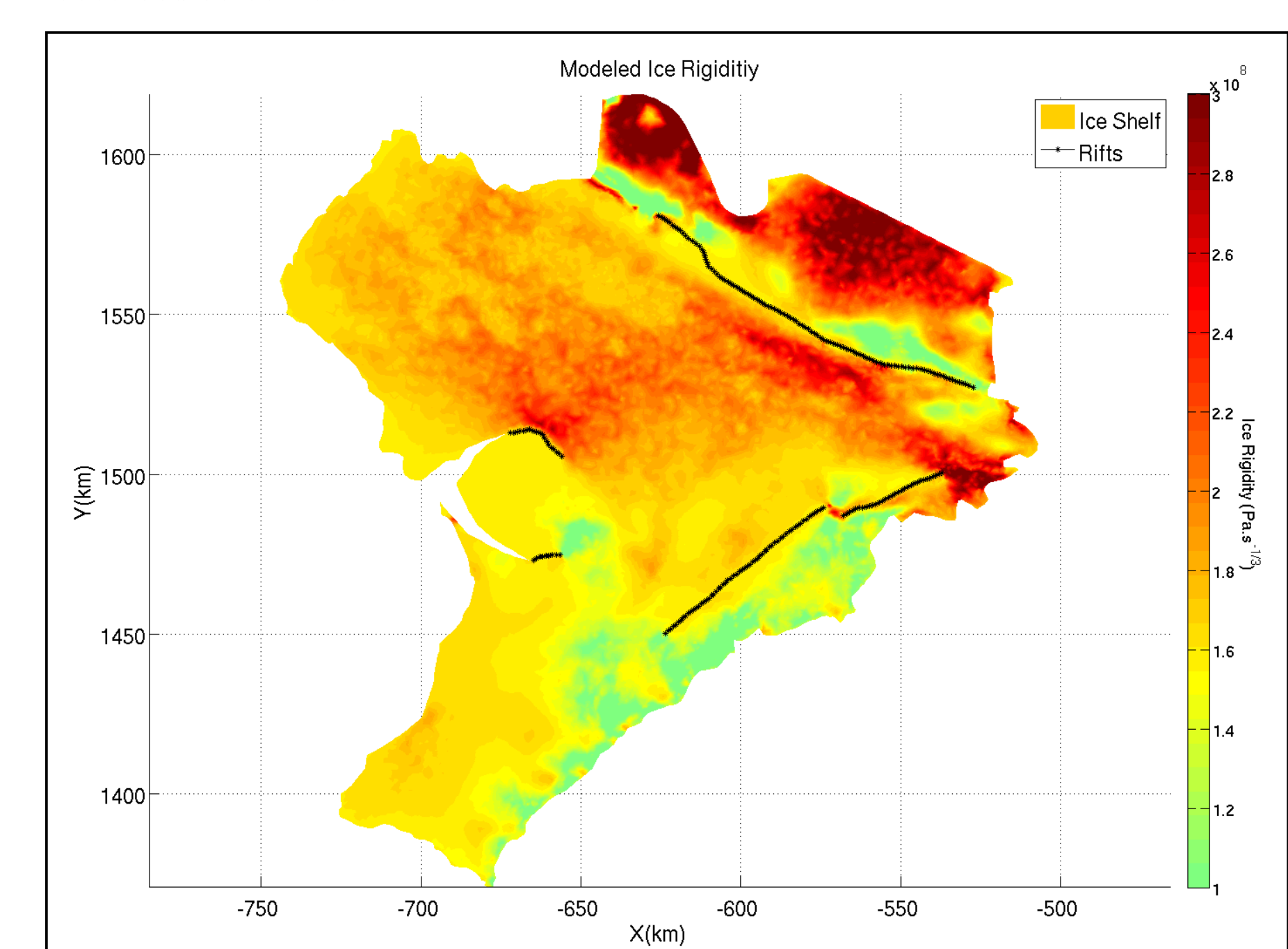
Across rift velocities, using control method results. Gray rifts are opening, black rifts are closing (faulting). For closing areas, across fault velocities are tangential to the fault line, and linear friction is applied (friction coefficient =  $10^{-11}$  Pa/s). For opening rifts, across rift velocities rotate along the rift axis. Rifts are filled with water. High opening rates are modeled for ice front rifts, which corresponds to currently observed propagation in the area.



Modeled ice rigidity using control methods and InSAR velocity (Rignot 2008), using a continuum mechanics model across rifts. The control method is compensating for the ice discontinuity by decreasing ice rigidity downstream and upstream of rifts. Opening rates cannot be correctly captured by continuum mechanics, leading to unrealistic ice rigidity. The control method also tries to accommodate the tip singularities of ice front rifts penetrating inside the shelf.



Modeled velocity (upper left), observed velocity (upper right) and relative difference (in %) between the two. Fit is good with observations, including around rifts/faults (<10 %). Modeled velocity is still missing some of the rotation in the ice shelf, which explains biggest misfits near the ice front. In a narrow band (<2km) around the rifts/faults, misfit flares up (>30%), which is attributed to the difficult mapping of the exact position of a rift singularity. The position is determined using observed velocities.



Modeled ice rigidity using control methods and InSAR velocity (from Rignot 2008). Modeled rifts are marked in black. Ice rigidity does not exhibit singularities across rifts, usually seen when using continuum mechanics (Khazendar et al. 2007). Patterns in ice rigidity vary significantly near the ice front, and in the vicinity (~50km) of rifts/faults. Zones of weakness that were observed using continuum mechanics have disappeared (near eastern rift of Stanbcomb-Wills).

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